ANALYSIS OF SINGLE-LAYERED MULTICONDUCTOR TRANSMISSION LINES USING THE FOURIER TRANSFORM AND MODE-MATCHING TECHNIQUES

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ABSTRACT: *The Fourier transform and mode-matching techniques are applied in the analysis of single-layered multiconductor transmission lines with finite strip thickness. Based on a quasi-static approach, the problem is formulated to obtain simultaneous equations for the modal coefficients of the potential distributions between transmission lines. Residue calculus is applied to represent the potential distributions in convergent series form. Analytical closed-form expressions for the self and mutual capacitances are developed. Numerical computations are performed to compare the validity of the proposed with other published data.* © 2003 Wiley Periodicals, Inc. Microwave Opt Technol Lett 36: 315–317, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10752

Key words: *multiconductor transmission lines; capacitance; Fourier transform; mode-matching technique*

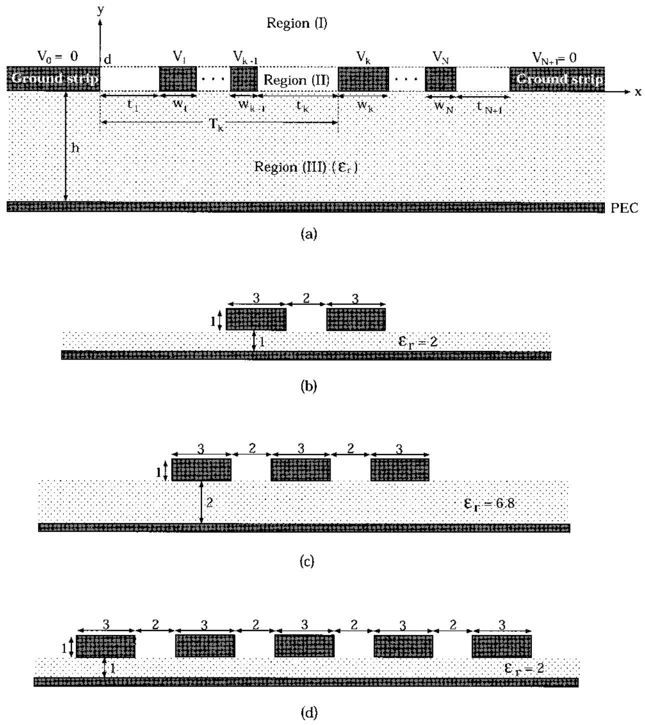

Figure 1 (a) Geometry of single-layered multiconductor transmission lines; (b) two-conductor transmission lines; (c) three-conductor transmission lines; (d) five-conductor transmission lines

1. INTRODUCTION

Recently, as the signal speed on printed circuit board (PCB) has dramatically increased, the importance of accurate extraction of circuit parameters has become more essential to the analysis of signal integrity on PCB. During the last decades, many studies for the analysis of planar transmission lines have been performed by using quasi-TEM approaches such as the boundary element method (BEM) [1], finite-element method (FEM) [2], finite-difference method (FDM) [3], spectral domain analysis (SDA) [4], and method of lines (MoL) [5] as well as full-wave analyses such as the finite-difference time-domain (FDTD) [6] and partial element equivalent circuit (PEEC) methods [7]. If the conductor cross-sectional dimensions of interconnect structures are small compared to the wavelength of the highest significant frequency in the signal, generally within a few GHz, the quasi-TEM approach is sufficiently applicable and is computationally efficient for the analysis of multiconductor transmission lines. The approximation based on the quasi-TEM approach is thus good enough for many practical situations.

In this paper, we present the Fourier transform and mode-matching techniques for evaluating the quasi-static parameters of single-layered multiconductor transmission lines with finite strip thickness. This method has been successfully applied to electrostatic and electrodynamic problems and its applicability for getting accurate and efficient modal solutions has been validated in [8, 9]. The Fourier transform and mode-matching techniques used in this paper provide a much simpler analytical modal solution than other previous numerical methods for extracting the capacitances of multiconductor transmission lines. It must be noted that the thickness of all conductors placed on the same layer should be taken as equal, which is a shortcoming of our method. In the next sections, the simultaneous equations to the modal coefficient are derived and the self and mutual capacitances between multiconductor transmission lines are expressed as analytical series forms in terms of modal coefficients. Numerical computations are performed to compare with other numerical data for accurate verification of the proposed method.

2. FIELD REPRESENTATION AND BOUNDARY CONDITIONS

Consider a single-layer multiconductor transmission line with a finite strip thickness, shown in Figure 1(a). In this quasi-static approximation, Laplace's equation for the electrostatic potential Φ is applicable to the problem analysis. In regions (I) ($y > d$) and (III) ($-h < y < 0$), the electrostatic potentials take the following forms:

$$\Phi^I(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\Phi}^I(\zeta)e^{-i\zeta x-|\zeta|(y-d)}d\zeta, \tag{1}$$

$$\Phi^{III}(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\Phi}^{III}(\zeta)\sinh \zeta(y+h)e^{-i\zeta x}d\zeta, \tag{2}$$

where $\tilde{\Phi}^I(\zeta)$ and $\tilde{\Phi}^{III}(\zeta)\sinh(\zeta h)$ are the Fourier transforms of $\Phi^I(x, d)$ and $\Phi^{III}(x, 0)$, given by $\tilde{\Phi}^I(\zeta) = \int_{-\infty}^{\infty} \Phi^I(x, d)e^{i\zeta x}dx$ and $\tilde{\Phi}^{III}(\zeta)\sinh(\zeta h) = \int_{-\infty}^{\infty} \Phi^{III}(x, 0)e^{i\zeta x}dx$, respectively. Using the superposition, the total potentials in region (II) ($T_k - t_k < x < T_k (k = 1, 2, \cdots, N+1)$, $0 < y < d$) are given by a sum of two solutions:

$$\Phi_k^{II}(x, y) = \sum_{m=1}^{\infty} [b_{mk}\cosh(a_{mk}y) + c_{mk}\sinh(a_{mk}y)]\sin[a_{mk}(x - T_k)] + \frac{V_k(x - T_k + t_k) + V_{k-1}(T_k - x)}{t_k}, \quad (3)$$

where $a_{mk} = (m\pi)/t_k$ $(m = 1, 2, 3, \cdots)$, $T_k = \sum_{j=1}^{k}(w_{j-1} + t_j)(k = 1, 2, \cdots, N + 1)$, and $w_0 = 0$. w_j is the width of the j^{th} microstrip line and t_j is the separation length between the $(j - 1)^{\text{th}}$ and the j^{th} microstrip lines. V_k is the voltage (or potential) of k^{th} strip $(V_0 = V_{N+1} = 0)$.

In order to determine the unknown modal coefficients b_{mk} and c_{mk}, we enforce the boundary conditions at $y = d$

$$\Phi^{I}(x, d) = \begin{cases} \Phi_k^{II}(x, d), & T_k - t_k < x < T_k(k = 1, 2, \cdots, N + 1) \\ V_k, & T_k < x < T_k + w_k(k = 1, 2, \cdots, N) \\ 0, & x < 0,\ x > T_{N+1} \end{cases} \quad (4)$$

and

$$\left.\frac{\partial \Phi^{I}}{\partial y}\right|_{y=d} = \left.\frac{\partial \Phi_k^{II}}{\partial y}\right|_{y=d}, \quad T_k - t_k < x < T_k(k = 1, 2, \cdots, N + 1). \quad (5)$$

Applying the Fourier transform to Eq. (4), we obtain

$$\tilde{\Phi}^{I}(\zeta) = \sum_{k=1}^{N+1} \left\{ \sum_{m=1}^{\infty} [b_{mk}\cosh(a_{mk}d) + c_{mk}\sinh(a_{mk}d)]F_m(\zeta t_k)e^{i\zeta T_k} + \frac{(V_k - V_{k-1})}{t_k} \frac{[e^{i\zeta(T_k} - e^{i\zeta(T_k - t_k)}]}{\zeta^2} \right\}, \quad (6)$$

where

$$F_m(\zeta t_k) = \frac{a_{mk}[1 - (-1)^m e^{-i\zeta t_k}]}{\zeta^2 - a_{mk}^2}. \quad (7)$$

Substituting $\tilde{\Phi}^{I}(\zeta)$ into Eq. (5), multiplying Eq. (5) by $\sin a_{pl}(x - T_l)$, and performing integration from $(T_l - t_l)$ to T_l, we obtain

$$\sum_{k=1}^{N+1} \left\{ \sum_{m=1}^{\infty} [b_{mk}\cosh(a_{mk}d) + c_{mk}\sinh(a_{mk}d)]I_{mpkl}^{(1)} + \frac{(V_k - V_{k-1})}{t_k} S_{pkl}^{(1)} \right\} = -p\pi^2[b_{pl}\sinh(a_{pl}d) + c_{pl}\cosh(a_{pl}d)], \quad (8)$$

where

$$I_{mpkl}^{(1)} = \int_{-\infty}^{\infty} |\zeta| F_m(\zeta t_k) F_p(-\zeta t_l) e^{i\zeta(T_k - T_l)} d\zeta, \quad (9)$$

$$S_{pkl}^{(1)} = \int_{-\infty}^{\infty} \frac{[1 - e^{-i\zeta t_k}]}{|\zeta|} F_p(-\zeta t_l) e^{i\zeta(T_k - T_l)} d\zeta, \quad (10)$$

and $p = 1, 2, 3 \ldots$, $l = 1, \ldots, N + 1$. It is possible to evaluate the integrals $I_{mpkl}^{(1)}$ and $S_{pkl}^{(1)}$ into a fast convergent series, based on residue calculus and mathematical tables [10]. Similarly, by using the boundary conditions at $y = 0$, we obtain

$$\varepsilon_r \sum_{k=1}^{N+1} \left\{ \sum_{m=1}^{\infty} b_{mk} I_{mpkl}^{(2)} + \frac{(V_k - V_{k-1})}{t_k} S_{pkl}^{(2)} \right\} = p\pi^2 c_{pl}, \quad (11)$$

where

$$I_{mpkl}^{(2)} = \int_{-\infty}^{\infty} \frac{\zeta}{\tanh(\zeta h)} F_m(\zeta t_k) F_p(-\zeta t_l) e^{i\zeta(T_k - T_l)} d\zeta, \quad (12)$$

$$S_{pkl}^{(2)} = \int_{-\infty}^{\infty} \frac{[1 - e^{-i\zeta t_k}]}{\zeta \tanh(\zeta h)} F_p(-\zeta t_l) e^{i\zeta(T_k - T_l)} d\zeta. \quad (13)$$

Note that Eqs. (8) and (11) constitute a system of simultaneous equations for the unknown modal coefficients b_{mk} and c_{mk}.

3. EVALUATION OF CAPACITANCE AND NUMERICAL EXAMPLES

The simultaneous equations in the previous section are solved for the unknown modal coefficients b_{mk} and c_{mk}. Then, the mutual capacitance per unit length of l^{th} conductor, due to the unit voltage of j^{th} conductor, will be given by

$$C_{lj} = \frac{Q_l}{V_j(=1\text{ V})} = -\varepsilon \int_S \frac{\partial \Phi(x, y)}{\partial n} dS = -\text{sgn}(l - k) \frac{\varepsilon_0}{\pi} \sum_{k=1}^{N+1} \left\{ \sum_{m=1}^{\infty} [b_{mk}\cosh(a_{mk}d) + c_{mk}\sinh(a_{mk}d)] \cdot [J(a_{mk}|T_k - T_l - w_l|) - J(a_{mk}|T_k - T_l|) - (-1)^m\{J(a_{mk}|T_k - T_l - t_k - w_l|) - J(a_{mk}|T_k - T_l - t_k|)\}] + \frac{(V_k - V_{k-1})}{t_k} [|T_l + w_l - T_k|\ln|T_l + w_l - T_k| - |T_l - T_k|\ln|T_l - T_k| + |T_l - T_k + t_k|\ln|T_l - T_k + t_k| - |T_l + w_l - T_k + t_k|\ln|T_l + w_l - T_k + t_k|] \right\} + \varepsilon_0 \sum_{m=1}^{\infty} [b_{ml}\sinh(a_{ml}d) + c_{ml}\cosh(a_{ml}d) - c_{ml} - (-1)^m\{b_{m(l+1)}\sinh(a_{m(l+1)}d) + c_{m(l+1)}\cosh(a_{m(l+1)}d) - c_{m(l+1)}\}] + \varepsilon_0 \left[\frac{V_l - V_{l-1}}{t_l} - \frac{V_{l+1} - V_l}{t_{l+1}} \right] d$$

$$+ \operatorname{sgn}(l-k)\frac{\varepsilon_0\varepsilon_r}{h}\sum_{k=1}^{N+1}\left\{\sum_{m=1}^{\infty}\sum_{\substack{u=1\\ v=u\pi/h}}^{\infty}\frac{b_{mk}a_{mk}v}{(v^2+a_{mk}^2)}\left[e^{-v|T_k-T_l|}-e^{-v|T_k-T_l-w_l|}-(-1)^m\{e^{-v|T_k-t_k-T_l|}-e^{-v|T_k-t_k-T_l-w_l|}\}\right]\right.$$

$$\left.+(V_k-V_{k-1})\left[\frac{w_l}{2}+\sum_{\substack{u=1\\ v=u\pi/h}}^{\infty}\frac{1}{v^2t_k}\left(e^{-v|T_k-T_l-w_l|}-e^{-v|T_k-T_l|}-e^{-v|T_k-t_k-T_l-w_l|}+e^{-v|T_k-t_k-T_l|}\right)\right]\right\}, \quad (14)$$

TABLE 1 Self and Mutual Capacitances of Two-Conductor Transmission Lines as Shown in Fig. 1(b) (Unit = pF/m)

	Our Method	BEM [1]	FEM [2]	MoL [5]
C_{11}	89.65	91.65	93.69	88.59
C_{12}	−8.19	−8.22	−8.27	−8.23

where

$$J(t) = \begin{cases} \sin(t)\mathrm{ci}(t) - \cos(t)\left[\mathrm{si}(t) + \dfrac{\pi}{2}\right] & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases}, \quad (15)$$

and $\operatorname{sgn}(x)$ is the signum function with $\operatorname{sgn}(0) = 1$. If $l = j$, the self capacitance can also be obtained.

For numerical examples, three types of single-layered multiconductor transmission lines are considered, as depicted in Figure 1(b–d). These were previously used in [1, 2, 5, 12]. The capacitance matrices are compared with those computed by other numerical methods in Tables 1, 2, and 3, thus indicating acceptable agreements.

5. CONCLUSION AND FUTURE WORK

The characteristics of a single-layered multiconductor transmission line with a finite strip thickness is studied using the Fourier transform and mode-matching technique based on quasi-static approach. The simple and analytical closed-form expressions for the self and mutual capacitances are developed. The results of capacitance matrices, which are useful for the analysis of crosstalk between high-speed signal traces on the printed circuit board, are compared with other published data for the validity of the proposed method. The extraction of capacitance matrices for two-dimensional multiconductor interconnects in a multilayered dielectric medium will be also analyzed with the proposed method as a future work.

TABLE 2 Self and Mutual Capacitances of Three-Conductor Transmission Lines as Shown in Fig. 1(c) (Unit = pF/m)

	Our Method	MoL [5]
C_{11}	155.70	157.00
C_{12}	−15.64	−15.61
C_{13}	−0.831	−1.326
C_{21}	−15.69	−15.61
C_{22}	160.50	160.10

TABLE 3 Self and Mutual Capacitances of Five-Conductor Transmission Lines as Shown in Fig. 1(d) (Unit = pF/m)

	Our Method	Bernal [12]
C_{11}	89.660	93.668
C_{12}	−8.110	−8.453
C_{13}	−0.795	−0.809
C_{21}	−0.319	−0.345
C_{22}	92.173	95.329
C_{23}	−7.962	−8.318
C_{24}	−0.730	−0.758
C_{33}	92.145	95.341

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