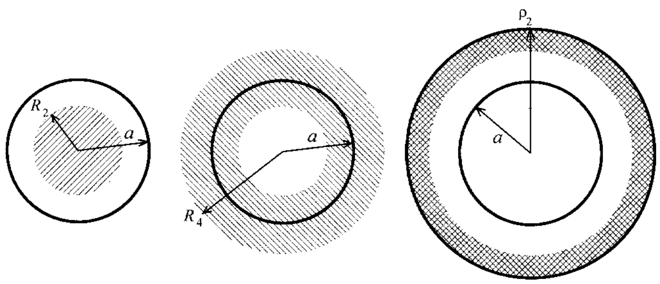

(a) (b) (c)

Fig. 1. The ray representation of the dominant HE_{11} mode of a circular HDW: the separation of the space around the HDW axis into the characteristic regions (indicated by different cross hatchings) concentric on this axis. (a) The region for rays associated with the plane homogeneous wave. (b) The region for rays associated with the plane homogeneous wave plus the mode LP_{21}. (c) The region for rays associated with the plane homogeneous wave plus the mode LP_{21} plus the mode LP_{41}.

for modeling the scattering of millimeter and submillimeter plane homogeneous waves by a physical object.

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TM Scattering from Slits in Thick Parallel Conducting Screens

Jae W. Lee and Hyo J. Eom

***Abstract*— TM-wave scattering from slits in thick parallel conducting screens is analyzed using the Fourier transform. The simultaneous equations for the modal coefficients are formed in terms of convergent series. Numerical computations are performed to illustrate the behavior of transmission and scattering from the slits. High-frequency solutions for scattering and transmission are obtained in approximate closed forms.**

***Index Terms*—Electromagnetic scattering.**

Scattering from slits in two parallel conducting screens was studied in [1]–[4] to understand mutual coupling between two slits. The present letter is a continuation of [5] where scattering from a single slit in a thick conducting plane was considered. Using the Fourier transform and mode matching, we obtain a solution in a convergent series form, which is numerically efficient and reduces to simple forms in low- and high-frequency limits.

In region (I) (air, $z > \beta^0 = 0$), a TM wave $H_y^i(x,z)$ is incident on a slit (width: $2a^0$; depth: d^0) in a thick perfectly conducting screen (see Fig. 1). Region (II) $(-\beta^l - d^l < z < -\beta^l, \alpha^l - a^l < x < \alpha^l + a^l$, relative permittivity $\epsilon_{r\text{II}})$ and region (IV) $(-\beta^{l+1} - d^{l+1} < z < -\beta^{l+1}, \alpha^{l+1} - a^{l+1} < x < \alpha^{l+1} + a^{l+1}$, and relative permittivity $\epsilon_{r\text{IV}})$, respectively, denote the lth slit and $(l+1)$th slit filled with lossy materials. Region (III) $(-\beta^{l+1} < z < -\beta^l - d^l$, and relative permittivity $\epsilon_{r\text{III}})$ denotes the lossless dielectric slab bounded by the conducting screens. The wave number in region (I) is $k = \omega\sqrt{\mu\epsilon_0}$ and a time-harmonic factor $e^{-i\omega t}$ is suppressed throughout. In region (I), the total H field consists of the incident, reflected, and scattered fields as

$$H_y^i(x,z) = e^{ik_x x - ik_z z} \tag{1}$$

$$H_y^r(x,z) = e^{ik_x x + ik_z z} \tag{2}$$

$$H_y^s(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{H}_y{}^s(\zeta) e^{-i\zeta x + ik' z}\, d\zeta \tag{3}$$

where $k_x = k\sin\theta$, $k_z = k\cos\theta$, and $k' = \sqrt{k^2 - \zeta^2}$. In regions (II), (III), (IV), and (V) the total fields are

$$H_y^{\text{II}}(x,z) = \sum_{m=0}^{\infty} [b_m^l \cos\xi_m^l(z+\beta^l) + c_m^l \sin\xi_m^l(z+\beta^l)] \cdot \cos a_m^l(x + a^l - \alpha^l) \tag{4}$$

$$H_y^{\text{III}}(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\tilde{H}_y{}^{\text{III}+}(\zeta) e^{ik'(z+\beta^l+d^l)} + \tilde{H}_y{}^{\text{III}-}(\zeta) e^{-ik'(z+\beta^l+d^l)}] e^{-i\zeta x}\, d\zeta \tag{5}$$

$$H_y^{\text{IV}}(x,z) = \sum_{m=0}^{\infty} [b_m^{l+1} \cos\xi_m^{l+1}(z+\beta^{l+1}) + c_m^{l+1} \sin\xi_m^{l+1}(z+\beta^{l+1})] \cdot \cos a_m^{l+1}(x + a^{l+1} - \alpha^{l+1}) \tag{6}$$

$$H_y^V(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{H}^V(\zeta) e^{-i\zeta x - ik'(z+\beta^L+d^L)}\, d\zeta \tag{7}$$

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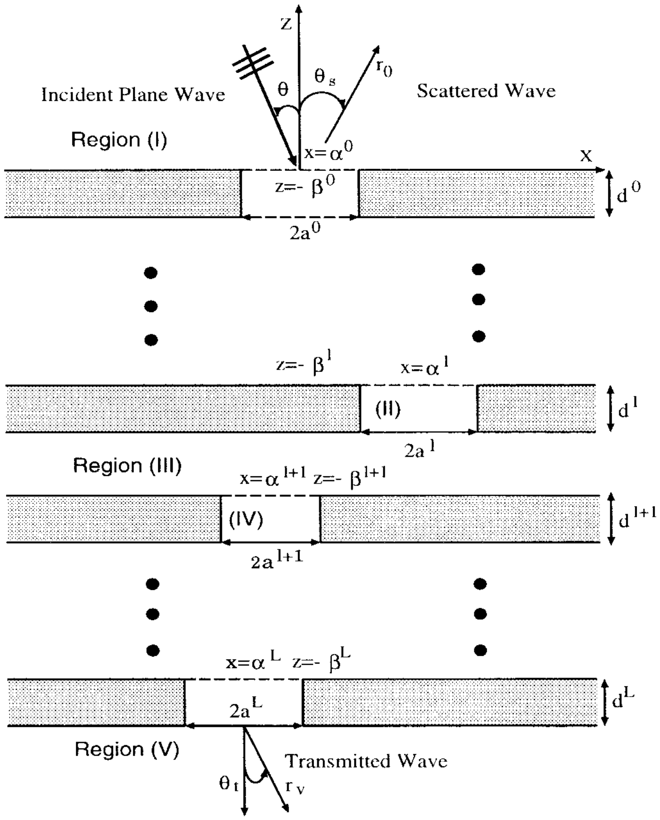

Fig. 1. Geometry of scattering from thick perfectly parallel conducting screens.

where $a_m^l = m\pi/(2a^l), \xi_m^l = \sqrt{k^2-(a_m^l)^2}$, and $k' = \sqrt{k^2-\zeta^2}$. The boundary condition at $z = 0$ gives

$$\frac{2ik_x}{[k_x^2-(a_n^l)^2]}[-(-1)^n e^{ik_x a^0} + e^{-ik_x a^0}]e^{ik_x \alpha^0} = \frac{i}{2\pi\epsilon_{r\mathrm{II}}}\sum_{m=0}^{\infty} \xi_m^0 c_m^0 I^0(k) + b_n^0 a^0 v_n \tag{8}$$

where

$$I^0(k) = \int_{-\infty}^{\infty} K_m^0(\zeta)K_n^0(-\zeta)\zeta^2(k')^{-1}\, d\zeta \tag{9}$$

$$= \frac{2\pi a^0 v_n}{\sqrt{k^2-(a_m^0)^2}}\delta_{mn} - [I_1^0(k) + I_2^0(k)] \tag{10}$$

$$K_m^l(\zeta) = \frac{[(-1)^m e^{i\zeta a^l} - e^{-i\zeta a^l}]}{[\zeta^2-(a_m^l)^2].} \tag{11}$$

δ_{mn} is the Kronecker delta and $v_0 = 2, v_n = 1$ $(n = 1, 2, 3, \cdots)$. The evaluation of $I^0(k)$ in terms of $I_1^0(k) + I_2^0(k)$ is given in [5]. The boundary conditions at $z = -\beta^l - d^l$ and $-\beta^{l+1}$ yield, respectively,

$$2\pi(b_n^l \cos\xi_n^l d^l - c_n^l \sin\xi_n^l d^l)a^l v_n = \sum_{m=0}^{\infty}\left[\frac{\epsilon_{r\mathrm{III}}}{\epsilon_{r\mathrm{II}}}\xi_m^l(b_m^l \sin\xi_m^l d^l + c_m^l \cos\xi_m^l d^l)I_{3nm} - \frac{\epsilon_{r\mathrm{III}}}{\epsilon_{r\mathrm{IV}}}\xi_m^{l+1}c_m^{l+1}I_{4nm}\right] \tag{12}$$

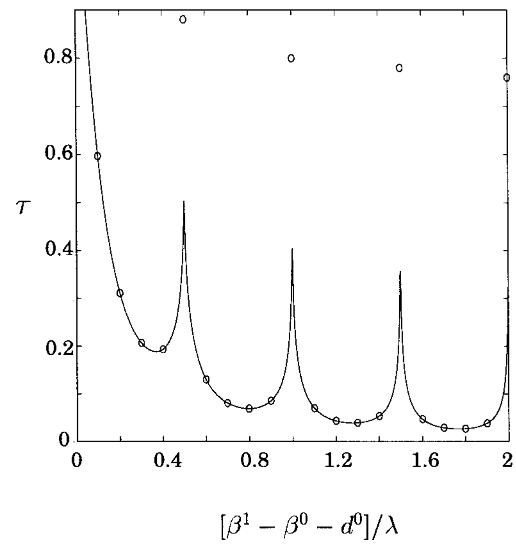

Fig. 2. Transmission coefficient, τ versus spacing between two screens, parameter: $a^0 = 0.5\lambda$, $a^1 = 0.1\lambda$, $d^0 = d^1 = 0\lambda$, $\alpha^0 = \alpha^1$, $\epsilon_{ri} = 1$ $(i = II,\ III,\ IV,\ V)$; —: present work; *oo*: [3].

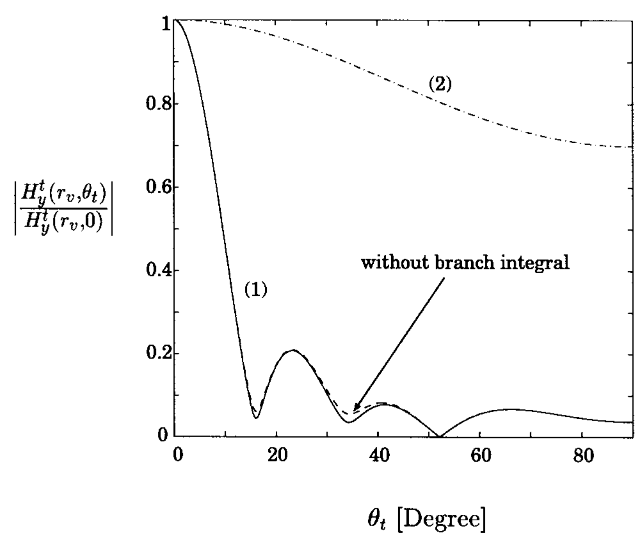

Fig. 3. Transmittal field pattern when $B^1 - d^0 - B^0 = 2/3\lambda, \alpha^0 = \alpha^1$, slit depths $(d^0,\ d^1) = 0$, and $\epsilon_{ri} = 1$, $(i = II,\ III,\ IV,\ V)$. (1) $\alpha^0 = a^1 = 1.9\lambda$. (2) $\alpha^0 = a^1 = 0.2\lambda$.

$$2\pi b_n^{l+1} a^{l+1} v_n = \sum_{m=0}^{\infty}\left[\frac{\epsilon_{r\mathrm{III}}}{\epsilon_{r\mathrm{II}}}\xi_m^l(b_m^l \sin\xi_m^l d^l + c_m^l \cos\xi_m^l d^l)I_{5nm} - \frac{\epsilon_{r\mathrm{III}}}{\epsilon_{r\mathrm{IV}}}\xi_m^{l+1}c_m^{l+1}I_{6nm}\right] \tag{13}$$

where

$$I_{3nm} = \int_{-\infty}^{\infty}\frac{\zeta^2 K_m^l(\zeta)K_n^l(\zeta)}{k'\tan k'(\beta^l + d^l - \beta^{l+1})}\, d\zeta \tag{14}$$

$$I_{4nm} = \int_{-\infty}^{\infty}\frac{\zeta^2 K_m^{l+1}(\zeta)K_n^l(\zeta)e^{i\zeta(\alpha^{l+1}-\alpha^l)}}{k'\sin k'(\beta^l + d^l - \beta^{l+1})}\, d\zeta \tag{15}$$

$$I_{5nm} = \int_{-\infty}^{\infty} \frac{\zeta^2 K_m^l(\zeta) K_n^{l+1}(\zeta) e^{i\zeta(\alpha^l - \alpha^{l+1})}}{k' \sin k'(\beta^l + d^l - \beta^{l+1})} \, d\zeta \tag{16}$$

$$I_{6nm} = \int_{-\infty}^{\infty} \frac{\zeta^2 K_m^{l+1}(\zeta) K_n^{l+1}(\zeta)}{k' \tan k'(\beta^l + d^l - \beta^{l+1})} \, d\zeta. \tag{17}$$

It is expedient to transform (14)–(17) into fast converging series, utilizing the residue calculus. The boundary condition at $z = -\beta^{l+1} - d^{l+1}$ yields

$$\begin{aligned} &2\pi(b_n^L \cos \xi_n^L d^L - c_n^L \sin \xi_n^L d^L) a^L \upsilon_n \\ &\quad = \frac{i\epsilon_{r\mathrm{V}}}{\epsilon_{r\mathrm{IV}}} \sum_{m=0}^{\infty} \xi_m^L (b_m^L \sin \xi_m^L d^L + c_m^L \cos \xi_m^L d^L) I^L(k) \end{aligned} \tag{18}$$

where

$$I^L(k) = \int_{-\infty}^{\infty} K_m^L(\zeta) K_n^L(-\zeta) \zeta^2 (k')^{-1} \, d\zeta \tag{19}$$

$$= \frac{2\pi a^L \upsilon_n}{\sqrt{k^2 - (a_m^L)^2}} \delta_{mn} - [I_1^L(k) + I_2^L(k)]. \tag{20}$$

The explicit expressions for $I_1^L(k)$ and $I_2^L(k)$ are given in [5]. Substituting (10) into (8) and (20) into (18) with (12) and (13), we obtain the simultaneous equations for b_m^l, c_m^l, b_m^{l+1}, and c_m^{l+1}. The transmission coefficient τ is defined as a ratio of the time-averaged power transmitted through the Lth slit to the time-averaged power incident on the zeroth slit. Fig. 2 describes the behavior of transmission coefficient τ as a function of slit separation when the slit widths are relatively small $2a^0 = 0.1\lambda, 2a^1 = 0.2\lambda$. It is seen that our solutions agree well with [3]. The resonance behavior in Fig. 2 can be explained by applying the transmission line model using a single TEM mode since the slits widths are small. Fig. 3 shows the far-zone angular transmitted-field patterns when the half slit widths a^0, a^1 are 1.9λ and 0.2λ. The angular transmitted field pattern for $a^0 = 0.2\lambda$ is much wider than that for $a^0 = 1.9\lambda$. When $a^0 = a^1 = 1.9\lambda$, the branch-cut contributions in $I^L(k)$ and $I^0(k)$ become negligible in high-frequency regime (i.e., $I_1^0(k) \approx I_2^0(k) \approx I_1^L(k) \approx I_2^L(k) \cong 0$), thus, significantly reducing the computational load.

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